Anchor Pair Selection in Unilateral TDoA Localization Topologies

Reza Zandian, Ulf Witkowski
Department of Circuits Design and Electronics
South Westphalia University of Applied Science
Soest, Germany
Email: {zandian.reza, witkowski}@fh-swf.de

***Abstract*— This paper addresses the pair selection problem of the unilateral time difference of arrival (TDoA) localization method. Two common concepts of pair selection are star form which uses a unique reference node for pairing all nodes and the chain form which links each node to its next available node. The problem of the star form is the possibility of occurring non-line of sight (NLOS) conditions between some anchors. The chain form has an issue with increasing variance of the noise as the number of anchors increases. A new hybrid form is proposed which avoids NLOS conditions and at the same time pertains the amount of noise at its minimum possible. Practical results confirm the superior performance of the proposed approach.**

Keywords*— *Localization, TDoA, UWB, Unilateral

I. Introduction

Among the localization approaches proposed so far, the time-based methods promise better accuracy and reliability in noisy and challenging conditions of the indoor environment. This is especially the case, when the ultra-wideband (UWB) localization systems are applied. UWB offers higher accuracy compared to other radio-based technologies due to its less sensitivity to multipath effect as a result of their large utilized bandwidth [1]. Possible connection topologies of time-based approaches are time of arrival (ToA) which extracts the range information from communications between only two nodes and time difference of arrival (TDoA) which performs the ranging between two anchors and one node. The TDoA approach is the main choice of designers when large number of nodes are involved in the network [2]. This is mainly due to the several features of this technique which are less signal interference, simplicity of the implementation, lower radio traffic, lower power consumption of the nodes and higher location update rate even though larger number of the nodes compared to ToA approach are involved [3].

The TDoA technique can be implemented in two different constellations [4, p. 192]. The first one is called unilateral in that the anchor nodes are transmitter and the mobile nodes are receiver. This constellation is useful for navigation and self-localization applications where the location data is required in the mobile node itself. The other constellation is multi-lateral in that the anchors are receiver and the mobile nodes are transmitter. This technique is more suitable for tracking and monitoring applications where the location data is collected in a central server [3]. The unilateral technique needs to manage a time delay between the transmission times of the anchors to avoid signal interference. This is achieved by defining a reference node which triggers the transmission and dictates the timing in the network. The unilateral approach does not require clock synchronization between the nodes, however, a clock drift management algorithm is required to equalize the clock paces of all the nodes [5].

One major issue in the design process of the unilateral TDoA method is the criteria used for anchor pair selection. In terms of location of the anchors, many solutions are proposed in the literature [6, p. 61] but the pair selection criteria has not been addressed so far. This paper, introduces two common methods of pair selection in unilateral TDoA approach as star form and chain form based on UWB devices. The details and properties of each form are discussed and a new pairing style is proposed as hybrid form which combines the two previously introduced options for the sake of improving the localization performance.

II. Unilateral TDoA topology

The TDoA localization technique estimates the location of a node using trilateration method. In this case, the time difference of signal arrival in the mobile node is measured which can be represented as a hyperbola line between the two nodes. The location of the mobile node is estimated by calculating the intersection point of the hyperbolas for all involved anchor nodes. This concept is shown in Fig. 1.

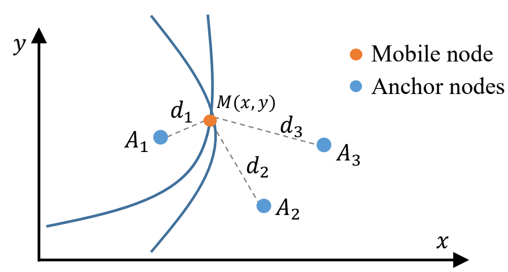

Fig. 1. Location estimation in the TDoA topology based on the intersection point of all hyperbolas

As mentioned before, anchor transmissions are performed sequentially with a certain time delay between transmissions. The transmission time of each anchor is calculated as:

$$t_{A_i tx} = t_{ref} + (i-1)t_{int}\ , \quad i = 2, \ldots, n \tag{1}$$

where t_{ref} is the time that the reference node has transmitted a signal, t_{int} is a fixed time interval between two sequential

https://doi.org/10.24355/dbbs.084-201807191327-0

transmissions and i indicates the index of the transmitting anchor. The differential time of arrival in the mobile node is the interval delay subtracted from the difference of the received time from the two anchors. This can be presented in general form as:

$$T_{i,j} = t_{A_j rx} - t_{A_i rx} - (i-j)t_{int} \ , \ i \neq j \tag{2}$$

where $T_{i,j}$ is the differential time between the node i and j where $i > j$, parameter $t_{A_j rx}$ is the signal arrival time of the anchor j at the mobile node and t_{int} is the interval delay.

III. Anchor pair selection concepts

In a localization system with many anchors, a concept is required to define which anchors should be paired in order to extract the differential time. A very common concept is to define the first anchor as reference and pair all the anchors with this anchor node. We call this pairing concept as star form. The connection structure of the star form for four anchors is demonstrated in Fig. 2.a.

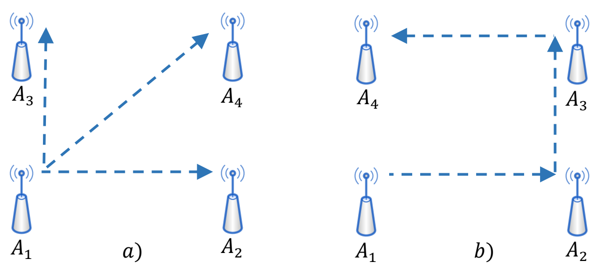

Fig. 2. a) Star form pairing concept of the anchors with anchor 1 as reference, b) Chain form of anchor pairing with linking anchors on the perimeter

The advantage of the star form is that, in all the anchors, same level of measurement noise can be observed as they are compared to a unique reference node. The next transmission time of the anchor n in the star form including the noise term w can be calculated using:

$$t_{A_n tx} = t_{ref} + (n-1)t_{int} + w_n \tag{3}$$

Applying this equation in (2), we can deduct the new term:

$$T_{i,1} = t_{A_1 rx} - t_{A_i rx} - (i-1)t_{int} + w_i' - w_1' \ , \quad i = 2, \dots, n \tag{4}$$

where parameter w_i' is the sum of the transmission noise in anchor i and the measurement noise in mobile node. In order to define a metric for the variance, the expectation value of the noise term should be calculated according to:

$$Q = E\left[\left(T_{i,1} - E(T_{i,1})\right)\left(T_{i,1} - E(T_{i,1})\right)^T\right] \ , \ i = 2, \dots, n \tag{5}$$

The solution of this equation leads to development of a covariance matrix with diagonal elements defined as:

$$Q = E[(w_i' - w_1')^2] = E\left[{w_i'}^2\right] + E\left[{w_1'}^2\right] - 2E[w_i' w_1'] \ , \ i = 2, \dots, n \tag{6}$$

Assuming that the noise of the anchors are uncorrelated, the term $E[w_i' w_1']$ will be zero. Defining σ^2 as the variance of the anchors, the final covariance matrix can be summarized as:

$$\begin{bmatrix} \sigma_2^2 + \sigma_1^2 & \sigma_1^2 & \dots & \sigma_1^2 \\ \sigma_1^2 & \sigma_3^2 + \sigma_1^2 & \dots & \sigma_1^2 \\ \dots & \dots & \dots & \dots \\ \sigma_1^2 & \sigma_1^2 & \dots & \sigma_n^2 + \sigma_1^2 \end{bmatrix} \tag{7}$$

As it can be seen, the non-diagonal terms are only correlated to the reference anchor. However, the variance of the diagonal terms are correlated to both anchors. If the noise characteristics in all the anchors are similar, it can be stated that the variance of differential time in star form is two times larger than the variance of noise observed from each anchor.

One major problem of the star form is that, some of the anchors are diagonally linked which means the line of sight of these anchors passes through the area where the mobile nodes are moving. This can cause a non-line of sight (NLOS) case which results in large error in the measurements. It is very hard to determine and compensate this problem in mobile node when the NLOS condition happens between the anchors. The chain form is another anchor pairing style which addresses this problem by linking the nodes that are located on the perimeter of the area. This is demonstrated in Fig. 2.b.

In the chain form, every anchor is paired with its next anchor. Therefore, the next transmission time of each anchor is dependent on the previous anchor. This can be defined for anchors 1 to n according to the following procedure:

$$t_{A_2 tx} = t_{ref} + t_{int} + w_1 \tag{8}$$

$$\begin{aligned} t_{A_3 tx} &= t_{A_2 tx} + t_{int} + w_2 \\ &= t_{ref} + 2t_{int} + (w_1 + w_2) \end{aligned} \tag{9}$$

$$\begin{aligned} t_{A_n tx} &= t_{A_{n-1} tx} + t_{int} + w_n \\ &= t_{ref} + (n-1)t_{int} + (w_1 + w_2 + \dots + w_n) \end{aligned} \tag{10}$$

As it can be seen, the noise term of each anchor includes the noise term of previous anchors. For this case, the general form of differential time and its covariance matrix can be defined as:

$$T_{i,(i-1)} = t_{A_{(i-1)} rx} - t_{A_i rx} - t_{int} + w'_{(i-1)} - w_i' \ , \ i = 2, \dots, n \tag{11}$$

$$Q = \begin{bmatrix} \sigma_2^2 + \sigma_1^2 & -\sigma_2^2 & \dots & -\sigma_n^2 \\ -\sigma_2^2 & \sigma_3^2 + \sigma_2^2 & \dots & -\sigma_n^2 \\ \dots & \dots & \dots & \dots \\ -\sigma_n^2 & -\sigma_n^2 & \dots & \sigma_n^2 + \sigma_{n-1}^2 \end{bmatrix} \tag{12}$$

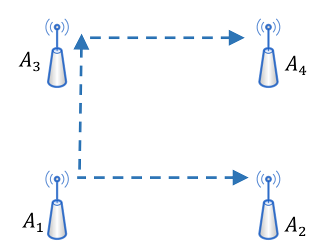

Fig. 3. Hybrid form of anchor pairing with 2 anchors connected according to the star form and one achor according to the chain form

In this case, each non-diagonal term is negatively correlated to the previous anchor and the variance of the diagonal terms are a parameter of the variance of previous anchor. This means, the

https://doi.org/10.24355/dbbs.084-201807191327-0

variance of the last anchor is n times larger than the variance of the first anchor where n is the total number of the anchors in the system. We propose a hybrid pairing style by connecting the anchors according to the star form once they are located on the perimeter of the area, and connecting them according to the chain form when they are diagonally located. This way the NLOS condition is avoided and the variance is pertained at minimum. The hybrid form is depicted in Fig. 3.

According to the hybrid form, the transmission time of the anchors can be defined for four anchors as follows:

$$t_{A_2tx} = t_{ref} + t_{int} + w_1 \tag{13}$$

$$t_{A_3tx} = t_{ref} + 2t_{int} + w_2 \tag{14}$$

$$\begin{aligned} t_{A_4tx} &= t_{A_3tx} + t_{int} + w_3 \\ &= t_{ref} + 3t_{int} + w_2 + w_3 \end{aligned} \tag{15}$$

As it can be seen, in this approach only the variance of the last anchor is increased while the variance of the third and second anchors are constant.

IV. Results and Disscussions

In order to evaluate the amount of the noise variances in practice, a setup including 4 UWB anchors and one mobile node based on each form is developed and the noise range of the received time stamps of the anchors at the mobile node are measured. The transmission interval was 50ms and the area of the filed was 60×40cm². The results for the case of star, chain and hybrid form are provided in Fig. 4, 5 and 6 respectively in cumulative distribution form (CDF). As it is visible in Fig. 4, the distribution of the noise for anchor 2 to 4 are in the same range. This proves the fact that in the star form the variance stays constant as it was stated in the covariance matrix in (7) this is however not the case for chain form as distribution is increasing as the number of anchors increases.

In the proposed hybrid form however, the anchor 4 has larger variance as this node is paired with the anchor 3. Anchor 2 and 3 have similar variances as they are paired according to the star form. In the chain form, the variance of each anchor increases similar to what happened to the anchor 4 in the hybrid form. In the end, it can be concluded that, although the proposed hybrid method has higher variance in one anchor, it has a better performance compared to the chain form and avoids the NLOS problem of the star form.

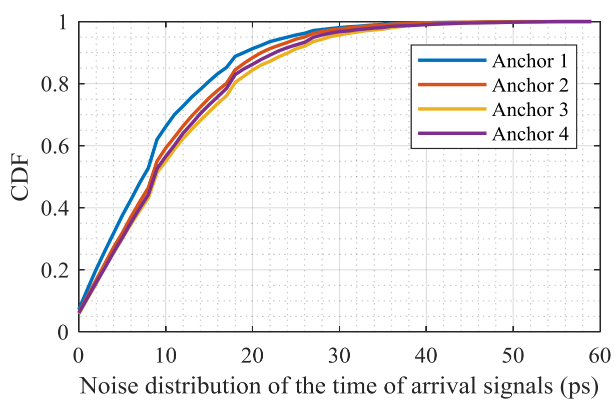

Fig. 4. The noise distribution of the received time stamps for the star form

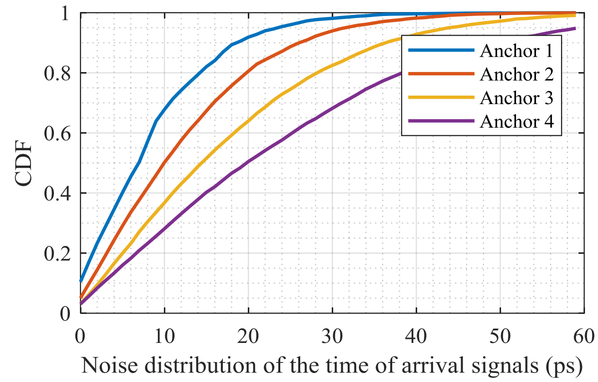

Fig. 5. The noise distribution in the received time stamps for the chain form

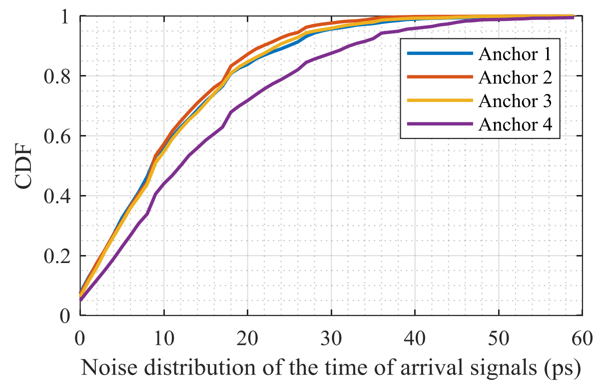

Fig. 6. The noise distribution of the received time stamps for the hybrid form

V. Conclusion

In this paper, common methods of the anchor pair selection in the unilateral TDoA topologies are introduced. The star form is the most common one which uses a common reference for all the nodes. Although this form has constant noise variance for all the anchors, it is disadvantaged from the NLOS problem for some of the anchors. In the chain form, this problem is solved but with the cost of increasing the noise variance linearly with the number of added anchors. The proposed hybrid method has better performance compared to the chain form as only one anchor has higher noise variance. Also the NLOS problem between the anchors is avoided which is an advantage. In case of 3D area with eight anchors, only two of the anchors have higher variance and five of them are paired according to the star form which limits the variance to be constant for those anchors.

References

[1] A. Yassin *et al.,* "Recent Advances in Indoor Localization: A Survey on Theoretical Approaches and Applications," *IEEE Commun. Surv. Tutorials*, vol. 19, no. 2, pp. 1327–1346, 2017.

[2] B. Xu, G. Sun, R. Yu, and Z. Yang, "High-Accuracy TDOA-Based Localization without Time Synchronization," *IEEE Trans. Parallel Distrib. Syst.*, vol. 24, no. 8, pp. 1567–1576, 2013.

[3] J. Kolakowski, J. Cichocki, P. Makal, and R. Michnowski, "An Ultra-Wideband System for Vehicle Positioning," *International Journal of Electronics and Telecommunications*, vol. 56, no. 3, p. 748, 2010.

[4] A. Bensky, *Wireless positioning technologies and applications*. Boston: Artech House, 2016.

[5] R. Zandian and U. Witkowski, "Robot self-localization in ultra-wideband large scale multi-node setups," in *2017 14th Workshop on Positioning, Navigation and Communications (WPNC)*, Bremen, Germany, 2017.

[6] Ł. Żwirełło, *Realization Limits of Impulse-Radio UWB Indoor Localization Systems*. Karlsruhe: KIT Scientific Publishing, 2013.

https://doi.org/10.24355/dbbs.084-201807191327-0